Wafer Probe Transducer Efficiency

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Abstract—**Experimental evidence is presented that shows the conventional expression relating the transducer efficiency of a two-port to measured scattering parameters is incorrect when the characteristic impedance at one of the ports is complex. This evidence is based on the measurement of the power from a microwave source transferred through a probe to a lossy coplanar waveguide. The conventional expression differs from the measurement by up to 20%. An alternative expression, accounting for the complex characteristic impedance, gives accurate results.**

I. Introduction

While the characteristic impedance of the travelling waves in common planar transmission lines, such as coplanar waveguide (CPW) and microstrip, is complex [1], the conventional expression governing the relationship between the scattering parameters and the microwave power transferred to these lines assumes it to be real. This may lead to errors in the determination of both signal and noise power transferred to and from planar circuits. In practice this occurs most often when transmission line behavior is dominated by resistive loss in thin film conductors, in which case the errors at low frequencies can be quite significant. Here, we compare measurements to both the conventional result and results from a theory [2], [3] that fully accounts for the complex characteristic impedance.

II. Direct Measurement of Transducer Efficiency

The experimental setup for the direct measurement of power transferred to an on-wafer load is shown in Fig. 1. Microwave power is transferred from the source through a microwave probe to a thermistor bead embedded in a short section of CPW fabricated on a gallium arsenide wafer. The source is defined to include not only a synthesizer but also the additional components which connect it to the probe. A 20-dB attenuator serves to equalize the reflection coefficients of the source in its on and off states.

A vector network analyzer calibrated to 50 Ω at a 2.4-mm coaxial test port was used to measure the reflection coefficients of the source (turned off) and the sensor head of a calibrated microwave power meter. The microwave power P_m delivered to the sensor head by the source was measured and the available power P_A from the source was determined in the usual way as

$$P_A = \frac{P_m |1 - \Gamma_S \Gamma_m|^2}{\left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_m|^2\right)}, \tag{1}$$

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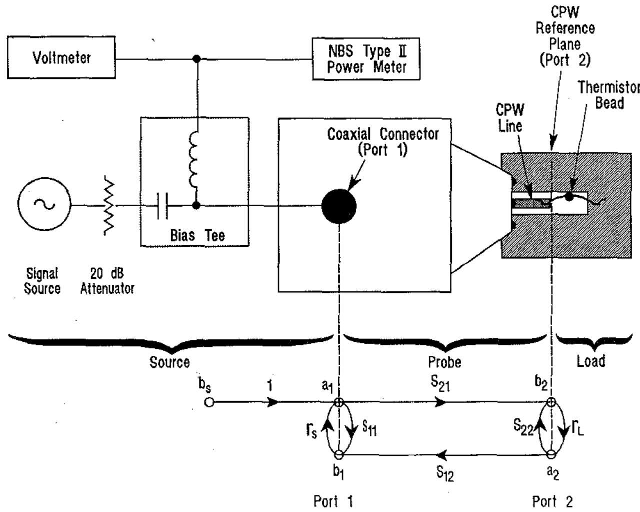

Fig. 1. Experimental setup. All quantities were measured with respect to either the coaxial connector of the microwave probe (port 1) or at the CPW reference plane coinciding with the center of the CPW thru line (port 2).

where Γ_S and Γ_m are the reflection coefficients of the source and the sensor head, respectively.

The microwave power transferred from the source to a thermistor bead mounted in the CPW was determined by a dc substitution method, implemented with an NBS Type II power meter [4]. The Type II power meter adjusts the dc-bias current so as to maintain the bead at a constant resistance and temperature both with and without power incident from the source. This allows the microwave power P_L dissipated in the thermistor bead to be determined from the difference of the dissipated dc powers, using

$$P_L = \frac{V_1^2 - V_2^2}{R_o}\left(1 - \frac{r}{R_o}\right), \tag{2}$$

where r is the resistance of the leads, probe head, and short section of CPW connecting the bias port to the thermistor bead, R_o is the total resistance at the bias port, and V_1 and V_2 are the measured voltages at the bias port when the microwave source is off and on, respectively. Equation (2) does not account for any RF–dc substitution error, which is usually small [5], and was neglected.

The fundamental quantity of interest is the transducer efficiency η of the microwave probes, including the short section of CPW between the probe tip and the CPW reference plane. This was determined from

$$\eta = \frac{P_L}{P_A}. \tag{3}$$

The transducer efficiency as defined by (3) is the equivalent of the transducer power gain described in [6] or the inverse of the transducer loss described in [7].

III. Scattering Parameter Measurement

The parameters S_{11}, S_{22}, and the product $S_{21}S_{12}$ (but not S_{21} and S_{12} independently) of the microwave probes and the short section of CPW that they contacted were measured by performing a two-tier calibration using the multiline thru-reflect-line (TRL) algorithm [8]. This procedure determines the scattering parameters relating the traveling waves, which have a complex characteristic impedance [1].

The CPW calibration set used in this procedure was constructed on the same wafer in which the thermistor bead was mounted. The CPW center conductors, of width of 73 μm, were separated from two 250-μm ground planes by 49-μm gaps in the 0.5-μm thick gold metallization. For purposes of performing this calibration, we used a thru line of 0.55-mm length, three lines of additional length 6.565 mm, 19.695 mm, and 40 mm, and two shorts offset 0.225 mm from the beginning of the line. The reflection coefficient Γ_L of the thermistor bead, which was placed as close as possible to the CPW reference plane, was measured with respect to this TRL calibration.

IV. Expression for Transducer Efficiency

The transducer efficiency may be determined not only by direct measurement but also in terms of the scattering parameters of the microwave probe, the reflection coefficients of the microwave source and thermistor bead, and the complex characteristic impedance of the lossy CPW.

Since the characteristic impedance Z_{o1} of the coaxial lines and the reference impedance at the coaxial ports of the microwave probe are essentially real, P_A is given by the usual expression [6], [7]

$$P_A = \frac{|b_s|^2}{1-|\Gamma_S|^2}, \tag{4}$$

where b_s is the source amplitude. The relationship between the source amplitude b_s, the traveling wave intensity b_2 at the CPW reference plane, and the scattering parameters of the probe is [6], [7]

$$\frac{b_2}{b_s} = \frac{S_{21}}{(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_S\Gamma_L}. \tag{5}$$

Since the characteristic impedance Z_{o2} of the CPW is complex, the net flow of power across that port is given in terms of the forward and backward traveling wave intensities a_2 and b_2 by [2]

$$P_L = -\left(|a_2|^2 - |b_2|^2 + 2\,\mathrm{Im}(a_2 b_2 *)\frac{\mathrm{Im}(Z_{o2})}{\mathrm{Re}(Z_{o2})}\right), \tag{6}$$

which can be expressed [2], [9]

$$P_L = |b_2|^2\left(1 - |\Gamma_L|^2 - 2\,\mathrm{Im}(\Gamma_L)\,\mathrm{Im}(Z_{o2})/\mathrm{Re}(Z_{o2})\right). \tag{7}$$

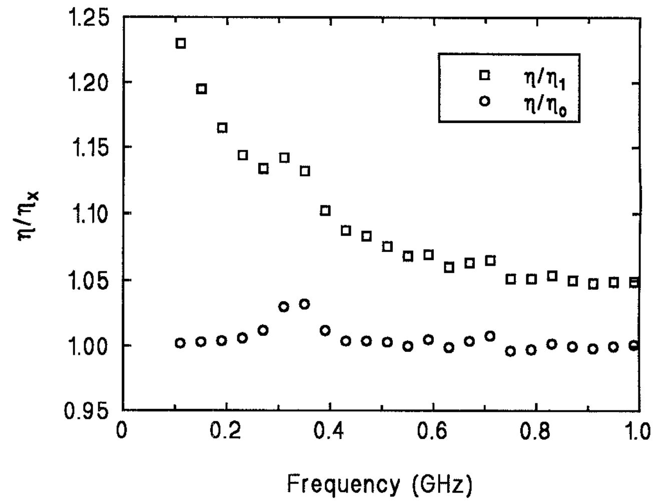

Fig. 2. The ratios η/η_1 and η/η_o. The increasing deviation at low frequency is related to the increasing magnitude of the phase angle of the characteristic impedance [1].

The calculated transducer efficiency η_o is thus,

$$\eta_o = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2 - 2\,\mathrm{Im}(\Gamma_L)\,\mathrm{Im}(Z_{o2})/\mathrm{Re}(Z_{o2})\right)}{|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_S\Gamma_L|^2}. \tag{8}$$

While (8) requires S_{21}, only the product $S_{21}S_{12}$ is determined by the de-embedding procedure. However, for linear reciprocal junctions, S_{21} and S_{12} are related by [2], [3]

$$\frac{S_{21}}{S_{12}} = \frac{K_1}{K_2}\frac{1-j\,\mathrm{Im}(Z_{o1})/\mathrm{Re}(Z_{o1})}{1-j\,\mathrm{Im}(Z_{o2})/\mathrm{Re}(Z_{o2})}, \tag{9}$$

where K_n, the reciprocity factors for each port, are close to 1 in quasi-TEM lines [3]. Assuming that $K_n = 1$, (9) determines S_{21} from the measured product $S_{21}S_{12}$.

V. Comparison to Measurement

In order to evaluate η_o, the phase angle of Z_{o2}, which is independent of the normalization chosen [2], [10], was determined using the method of [1]. The ratios of η to η_o for our experiment are plotted as circles in Fig. 2. The calculated transducer efficiency η_o predicts the measured efficiency η accurately, confirming the theory [2], [3], the assumptions in [1] used to find the characteristic impedance of the CPW, and the assumption $|K_n| \approx 1$.

Equation (8) simplifies to a more conventional form when the characteristic impedance Z_{o2} is real. Under this assumption, η_o reduces to the conventional expression for transducer efficiency [6], [7],

$$\eta_1 = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_S\Gamma_L|^2}, \tag{10}$$

where, for reciprocal junctions, the assumption that $S_{21} = S_{12}$ is invoked to determine $|S_{21}|^2$ from the measured $S_{21}S_{12}$. Because Z_{o2} is complex and because $S_{21} \neq S_{12}$, η_1 is not expected to agree with the measured value η. The ratios of the two quantities are represented as squares in Fig. 2. Here, η_1 deviates from η by over 20% at the low frequencies.

VI. CONCLUSION

Experimental results show that the usual expression for transducer efficiency does not hold when the characteristic impedance of the travelling waves at one of the ports is complex, as is often the case in on-wafer measurements. An expression which fully accounts for the complex characteristic impedance corresponds closely to measurements. Techniques for transforming to a real reference impedance and determining the pseudo-scattering parameters, thereby allowing the use of the conventional expression, are discussed in [1] and [2].

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